Centre Number | 1 | 2 | 5 |

CRANBROOK SCHOOL

Student Number | | | | | | | | |

2017

Assessment Task 3

Mathematics Extension 1

Reading time	5 minutes
Writing time	2 hours
Total Marks	70
Task weighting	40%

General Instructions

- Write using black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- The Reference sheet is on the last page of this booklet
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 4 writing booklets

Structure & Suggested Time Spent

Section I
Multiple Choice Questions

- Answer Q1 – 10 on the multiple choice answer sheet
- Allow about 20 minutes for this section

Section II
Extended response Questions

- Attempt all questions in this section in a separate writing booklet
- Allow about 100 minutes for this section

This paper must not be removed from the examination room

Disclaimer
The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

10 Marks

Allow about 20 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

---

1 The polynomial $P(x) = 2x^3 - kx^2 - 5x - 1$ leaves a remainder of 5 when divided by $x + 3$. Find the value of *k*.

(A) -5

(B) 5

(C) $-\frac{1}{5}$

(D) -3

2 The function $f(x) = e^{\sin x} - 1$ has a root between $x = 3$ and $x = 4$. In which of the following intervals does the root lie?

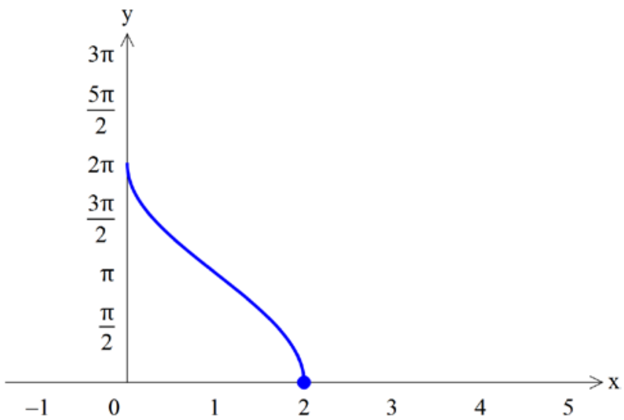
(A) $3 \le x \le 3.25$

(B) $3.25 \le x \le 3.5$

(C) $3.5 \le x \le 3.75$

(D) $3.75 \le x \le 4$

3 Consider the following graph.



Which of the following best represents the graph above?

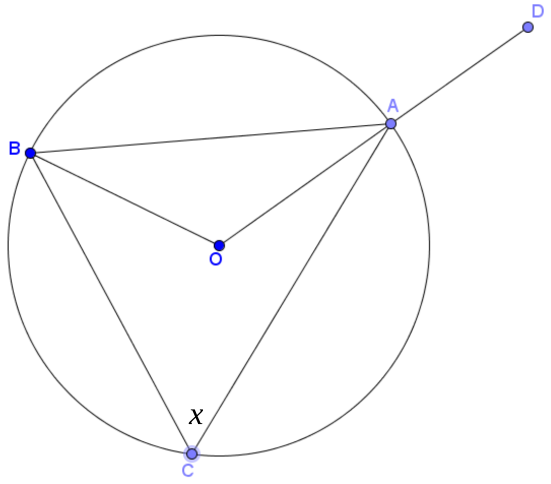
(A) $y = \frac{1}{2}\cos^{-1}(x-1)$

(B) $y = 2\cos^{-1}(x-1)$

(C) $y = \frac{1}{2}\sin^{-1}(x-1)$

(D) $y = 2\sin^{-1}(x-1)$

4 Consider the circle below with centre, O.



Given $\angle BCA = x$ and OAD is straight line, find $\angle BAD$ in terms of x.

(A) $\dfrac{\pi}{2} - x$

(B) $\dfrac{\pi}{2} + x$

(C) $2x$

(D) x

5 A particle with displacement x metres and velocity v metres/second, moves such that $v^2 = -9(x-3)(x+1)$. Find the maximum speed of the particle.

(A) $6\,\text{m/s}$

(B) $3\,\text{m/s}$

(C) $\pm 6\,\text{m/s}$

(D) $2\,\text{m/s}$

6 Which of the following is identically equal to $\sin x - 2\cos x$?

(A) $\sqrt{5}\cos(x + \tan^{-1} 2)$

(B) $\sqrt{5}\sin(x - \tan^{-1} \frac{1}{2})$

(C) $\sqrt{5}\cos(x + \tan^{-1} \frac{1}{2})$

(D) $\sqrt{5}\sin(x - \tan^{-1} 2)$

7 Which of the following is equal to $\tan\left[2\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right]$?

(A) x

(B) $2\sin x\cos x$

(C) $\dfrac{2x}{1-x^2}$

(D) $\dfrac{2x}{\sqrt{1+x^2}}$

8 Which of the following is equal to $\int \frac{3}{25+4x^2}\,dx$?

(A) $\frac{3}{10}\tan^{-1}\left(\frac{2x}{5}\right)+c$

(B) $3\tan^{-1}\left(\frac{2x}{5}\right)+c$

(C) $\frac{15}{8}\tan^{-1}\left(\frac{5x}{2}\right)+c$

(D) $3\tan^{-1}\left(\frac{5x}{2}\right)+c$

9 A cylinder, radius x and height $2x$, is increasing in volume at the rate of $0.1\text{cm}^3/\text{s}$.

Given the following information:

- The volume, V, is $V = 2\pi x^3$
- The surface area, S, is $S = 6\pi x^2$,

find the rate of change of the surface area when the radius is 5 centimetres.

(A) $0.1\ \text{cm}^2/\text{s}$

(B) $0.06\ \text{cm}^2/\text{s}$

(C) $0.01\ \text{cm}^2/\text{s}$

(D) $0.04\ \text{cm}^2/\text{s}$

10 Using the substitution $u = 1 - 2x$, the integral $\int_0^1 \frac{x}{\sqrt{1-2x}}\,dx$ can be expressed as:

(A) $\frac{1}{4}\int_{-1}^{1} \frac{1-u}{\sqrt{u}}\,du$

(B) $\frac{1}{4}\int_{1}^{-1} \frac{1-u}{\sqrt{u}}\,du$

(C) $\frac{1}{2}\int_{-1}^{1} \frac{1-u}{\sqrt{u}}\,du$

(D) $\frac{1}{2}\int_{1}^{-1} \frac{1-u}{\sqrt{u}}\,du$

END OF SECTION I

Section II

60 Marks

Allow about 100 minutes for this section

Answer questions 11-14 in separate booklets.

Question 11 **Start a new booklet** **15 Marks**

(a) Solve for x: $\dfrac{3}{2-x} \leq 1$ **3**

(b) Nine balls, each labelled 1 to 9, are to be lined up in a row.

(i) In how many ways can this be done? **1**

(ii) In how many ways can they be arranged if numbers 1 and 3 must be next to each other? **1**

(iii) In how many different ways can the balls be arranged so that that they alternate between even and odd numbers? **1**

For a number to be divisible by 4 the final two digits must form a multiple of 4.

(iv) If the balls are arranged randomly what is the probability that the number they form is divisible by 4? **2**

(c) The mass, W kilograms, of an African elephant can be approximated through the formula $W = 6000 - Ae^{-kt}$, where k is a constant and t is the elephant's age in years.

An elephant's mass at birth is 91 kilograms and at 5 years old is 2300 kilograms.

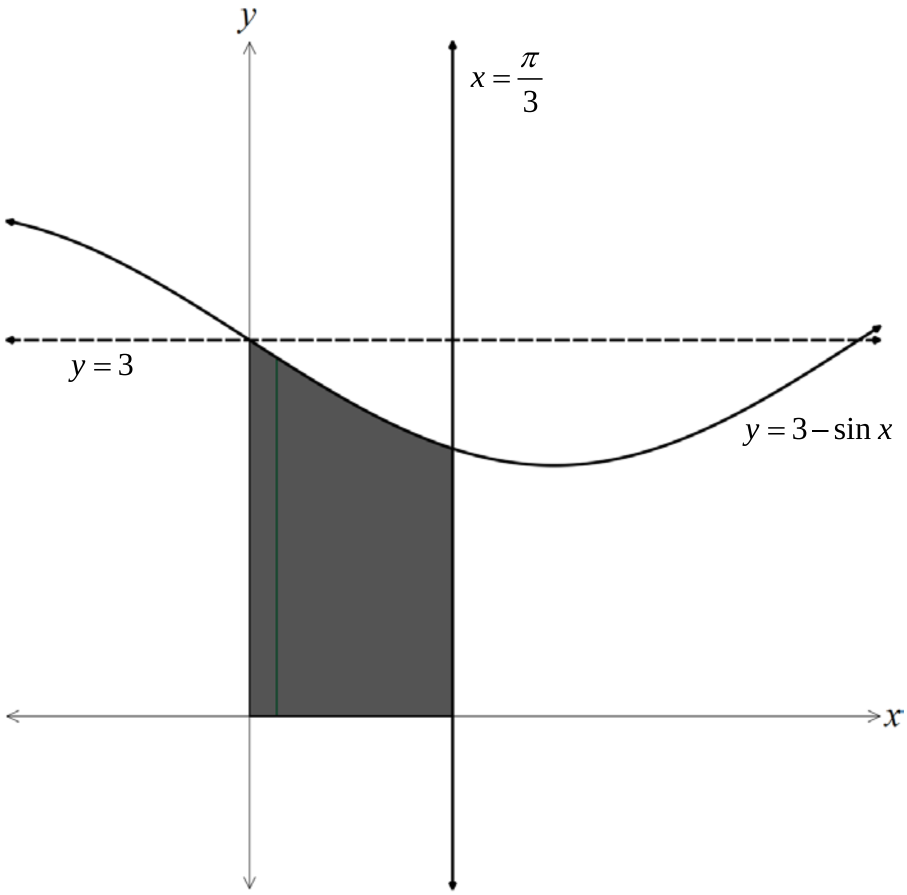
(i) Find the values A and k. **2**

(ii) At what rate is the mass of the elephant increasing when it is 20 years old? **1**

(iii) Draw a graph of the mass of the elephant against time. **1**

Question 11 continues on the next page

(d) The region bounded by the curve $y = 3 - \sin x$ and the x axis for $0 \leq x \leq \frac{\pi}{3}$ is shaded below. **3**



Find the volume of the solid formed if the shaded area is rotated around the x axis.

END OF QUESTION 11

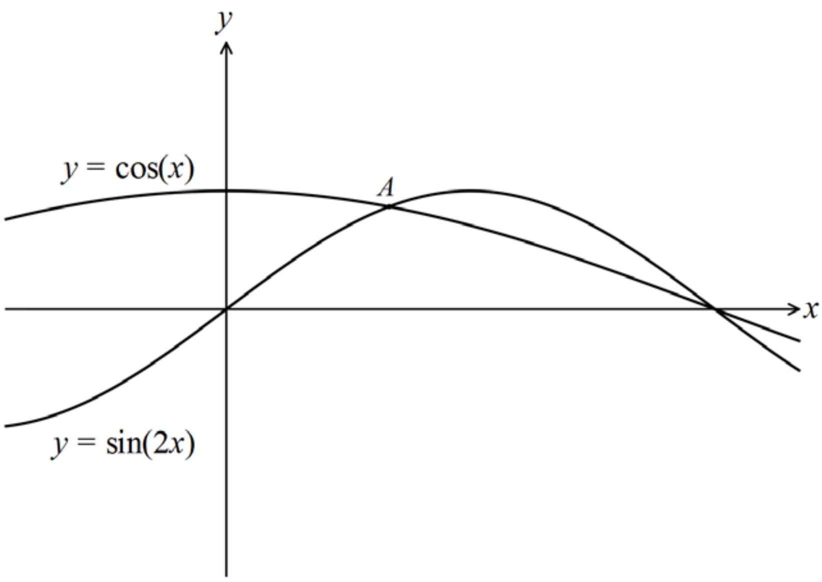
Question 12 **Start a new booklet** **15 Marks**

(a) Consider the function $f(\theta) = \sin\theta - \cos\theta$ over the domain $0 \le \theta \le 2\pi$.

(i) Use $t = \tan\dfrac{\theta}{2}$ to solve the equation $f(\theta) = 1$ **3**

(ii) Hence find θ such that $f(\theta) < 1$ **1**

(b) The curves $y = \sin 2x$ and $y = \cos x$ are shown below. In the domain $0 < x < \dfrac{\pi}{2}$ they intersect at point A.



Find the angle between the two curves at the point A. **4**

Question 12 continues on the next page.

(c) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$. The equation of the normal to the curve of the parabola at the point, P, is $x + py = 2ap + ap^3$ (You do not need to show this).

(i) Find the co-ordinates of the point Q where the normal at P meets the y axis. **1**

(ii) Show that the co-ordinates of the point R which divides PQ externally in the ratio $2:1$ are $R(-2ap, 4a + ap^2)$. **1**

(iii) Find the Cartesian equation of the locus of R and describe the locus in geometrical terms. **2**

(d) Newton's Method uses the tangent at a point near a root to give a better approximation for that root.

(i) Describe a situation in which Newton's method would NOT give a better approximation for a root. **1**

(ii) The curve $y = \sin^2 x + \ln(x-1)$ has a root near $x = 2.1$.

Use Newton's approximation to determine the next approximation. **2**

END OF QUESTION 12

Question 13 **Start a new booklet** **15 Marks**

(a) Hayley is a baseball coach. Each week she must pick a team of 9 players from her roster of 12. However, when picking a team she must pick a pitcher, a catcher, four infielders and 3 outfielders.

(i) How many different teams can she pick? **2**

(ii) Two of her players, Raul and Petra, don't work well together. How many different teams can Hayley pick if Raul and Petra can't be in the infield together or in the outfield together? **2**

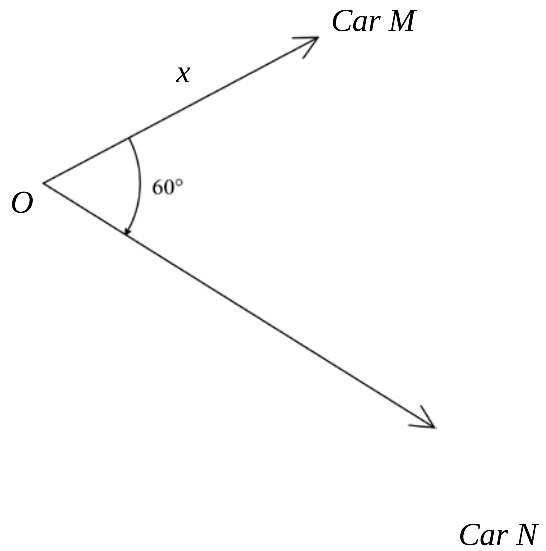
(b) A particle moves according to the equation: $v = (16 - x)$, where v is the velocity in metres per second, and x is the displacement in metres from the origin. The particle is initially at $x = 15$.

(i) Find the displacement, x as a function of time, t. **2**

(ii) Hence describe the motion of the particle as time goes on indefinitely. **1**

Question 13 continues on the next page.

(c) Two cars, M and N start moving from a point O at an angle of 60^o to each other. Car N travels at three times the speed of car M. Let the distance travelled by car M be x metres.

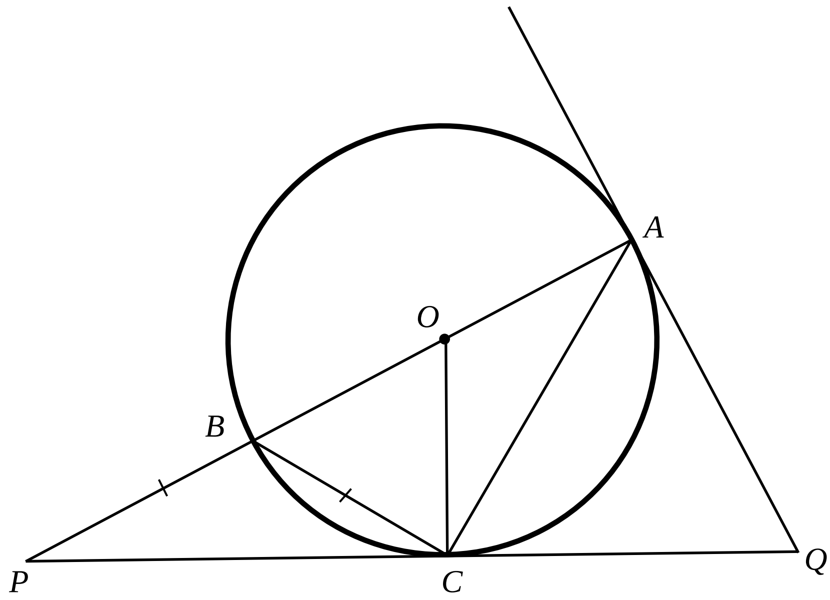


(i) Show that the area, A, of ΔOMN is $A=\dfrac{3\sqrt{3}x^2}{4}$. **1**

(ii) If car M is travelling at a speed of 15 m/s when $x=100$, what is the rate of change of the area A at this time? Leave your answer in exact form. **2**

Question 13 continues on the next page

(d) The circle centred at O has diameter AB. A point P on AB produced is chosen so that PC is a tangent to the circle at C and $BP = BC$. The tangents to the circle at A and C meet at Q.



(i) Prove $\angle BPC = 30^o$ **2**

(ii) Prove $\Delta POC \equiv \Delta QOC$ **3**

END OF QUESTION 13

Question 14 **Start a new booklet** **15 Marks**

(a) Consider the curve $y = \dfrac{9x^2 - 1}{x^4}$.

(i) Prove that the function is even. **1**

(ii) Find any stationary points and determine their nature. **2**

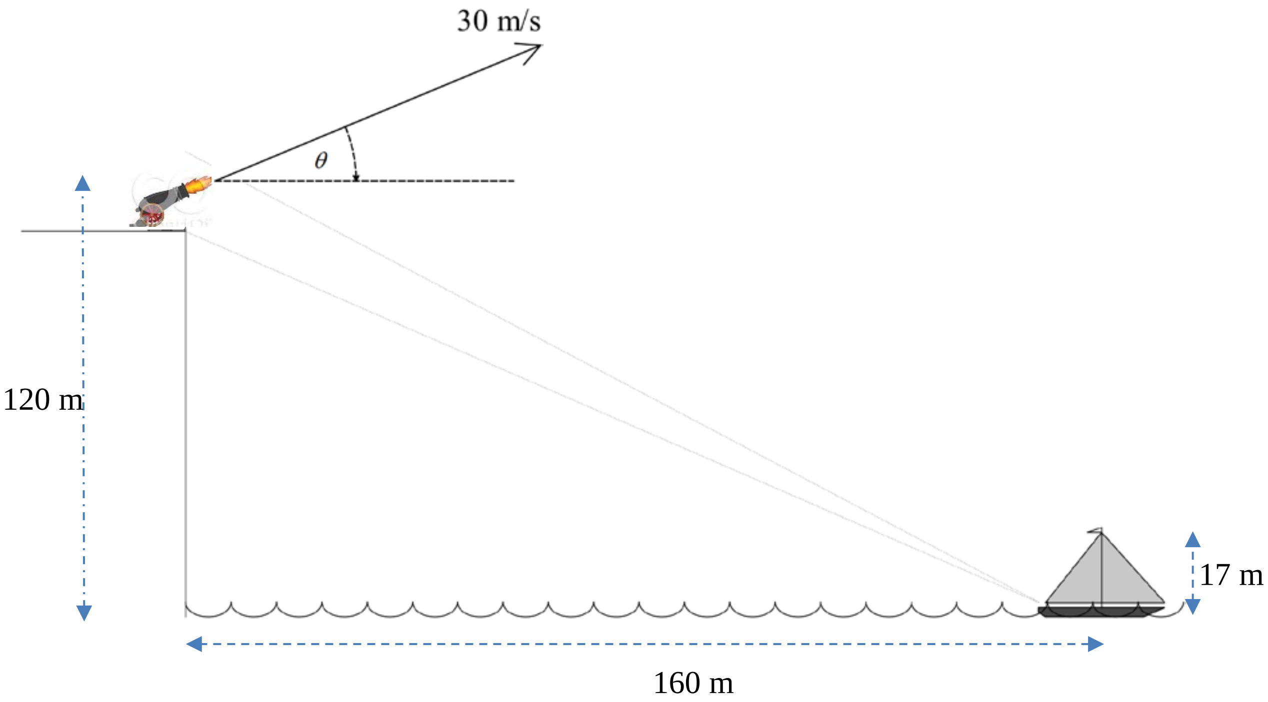
(iii) Find the points of inflexion. **2**

(iv) State any vertical asymptotes and the limit as x approaches infinity. **2**

(v) Sketch the curve showing all critical values. **2**

Question 14 continues on the next page

(b) A cannon ball is fired at a velocity of $v = 30\,\text{m/s}$ from a clifftop towards an enemy ship, 160 metres from the base of the cliff. The cannon is 120 metres above sea level. You may assume the acceleration due to gravity is $9.8\,\text{m/s}^2$.



(i) Show that the equations of motion are:

$x = 30t\cos\theta$ and $y = -4.9t^2 + 30t\sin\theta + 120$ **2**

(ii) In order to damage the ship, the cannon ball must hit the ship's mast, which is 17 metres above sea level and located at the centre of the ship (i.e. at 160 metres from the base of the cliff as indicated in the diagram).

Between what angles must the ball be fired in order to hit the ship's mast? **4**

END OF QUESTION 14

END OF EXAM

Year 12 Extension 1 Mathematics Trial Exam

Solutions

Q1 $P(-3) = 5$

$$P(-3) = 2(-3)^3 - k(-3)^2 - 5\times(-3) - 1$$

$$5 = -54 - 9k + 15 - 1$$

$$45 = -9k$$

$$k = -5 \quad \therefore \text{ Answer (A)}$$

Q2 $f(3) > 0$, $f(4) < 0$

$f(3.5) < 0$ $\quad \therefore$ Answer (A)

$f(3.25) < 0$

Q3 When $x = 0$, $y = 2\pi$

When $x = 0$

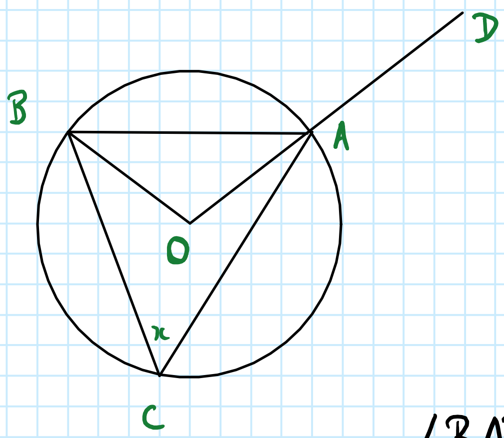
(A) $= \frac{\pi}{2}$

(B) $= 2\pi$ $\quad \therefore$ Answer (B)

(C) $= -\frac{\pi}{4}$

(D) $= -\pi$

Q4



$\angle BOA = 2x$ ($\angle$ subtended at the centre is twice the angle at circumference)

$\angle BAO = \dfrac{\pi - 2x}{2}$ ($\angle$ sum of isosceles ΔOBA)

$$\angle BAD = \pi - \left(\frac{\pi - 2x}{2}\right)$$

$$= \frac{\pi}{2} + x \quad \therefore \text{ Answer (B)}$$

Q5 $v^2 = -9(x-3)(x+1)$

$v = 0$ at the end points.

$x = 3$ & -1

$\therefore$ centre of motion @ $x = 1$

Max velocity occurs @ centre of motion

$v^2 = -9(-2)(2)$

$= 36$

$v = \pm 6 \text{ ms}^{-1}$ Max $V = 6 \text{ ms}^{-1}$ $\therefore$ Answer (A)

Q6 let $\sin x - 2\cos x = R\sin(x-\alpha)$

$= R\sin x\cos\alpha - R\cos x\sin\alpha$

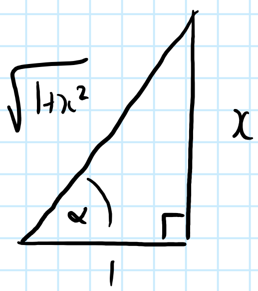
$\therefore$ $R\cos\alpha = 1$ $R\sin\alpha = 2$

$\therefore$ $R = \sqrt{5}$ $\tan\alpha = 2$ $\therefore$ Answer (D)

Q7 $\tan\left[2\sin^{-1}\dfrac{x}{\sqrt{1+x^2}}\right]$

$\tan 2\alpha$ where $\alpha = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$= \dfrac{2\tan\alpha}{1-\tan^2\alpha}$



$\therefore$ $\tan\alpha = x$

$= \dfrac{2x}{1-x^2}$

$\therefore$ Answer is (C)

Q8 $3\int \frac{1}{5^2+(2x)^2}\,dx$ $\therefore a=5$

$\frac{d}{dx}2x=2$

$\therefore = 3\left[\frac{1}{5}\tan^{-1}\left(\frac{2x}{5}\right)\div 2\right]+C$

$= \frac{3}{10}\tan^{-1}\left(\frac{2x}{5}\right)+C$ $\therefore$ Answer A

Q9 $\frac{dSA}{dt}=\frac{dV}{dt}\times\boxed{\frac{dSA}{dV}}$ $\frac{dSA}{dV}=\frac{dSA}{dx}\times\frac{dx}{dV}$

$\frac{dSA}{dx}=12\pi x$ $\frac{dV}{dx}=6\pi x^2$

$\therefore \frac{dSA}{dV}=\frac{2}{x}$

$\therefore \frac{dSA}{dt}=0.1\times\frac{2}{x}$

$=\frac{1}{5x}$ When $x=5$

$\frac{dSA}{dt}=\frac{1}{25}$

$=0.04$ cm^2/s $\therefore$ Answer D

Q10 $u = 1 - 2x$ When $x = 1$ $u = -1$

$\frac{du}{dx} = -2$ $x = 0$ $u = 1$

$$\int_0^1 \frac{x}{\sqrt{1-2x}}\,dx = \int_1^{-1} \frac{\frac{1-u}{2}}{\sqrt{u}} \times -\frac{du}{2}$$

$$= -\frac{1}{4}\int_1^{-1} \frac{1-u}{\sqrt{u}}\,du$$

We can flip the limits

$$= \frac{1}{4}\int_{-1}^{1} \frac{1-u}{\sqrt{u}}\,du$$

$\therefore$ Answer A

Answers to MC

1	2	3	4	5	6	7	8	9	10
A	A	B	B	A	D	C	A	D	A

(a) $$\frac{3}{2-x} \leq 1$$

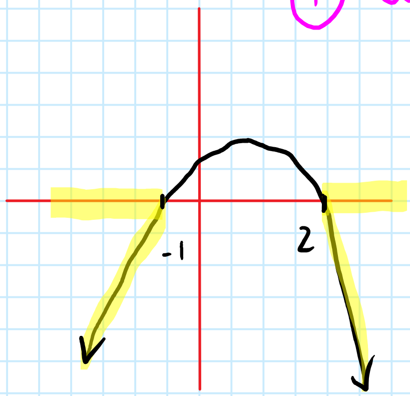
$$3(2-x) \leq (2-x)^2$$ ① × denominator squared

$$3(2-x) - (2-x)^2 \leq 0$$

$$(2-x)\left[3 - (2-x)\right] \leq 0$$

$$(2-x)(x+1) \leq 0$$

① Justification



$\therefore \quad x \leq -1 \quad , \quad x > 2$

① Correct answer

(b) (i) $9!$ ①

(ii) $$\underbrace{\underbrace{\boxed{1\ 3}}_{2 \text{ objects}}\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{8 \text{ objects}}$$

$\therefore \quad 2! \times 8!$ ①

5 odd
4 even

(iii) _ _ _ _ _ _ _ _ _

$\therefore \quad 4! \times 5!$ (1)

(iv) Multiples of 4 that can be formed

$\cancel{04}, \cancel{08}, 12, 16, \cancel{20}\ 24, 28, 32, 36, \cancel{40} \ldots$

No ball labelled zero & no 44 or 88

$\therefore \quad 2 + 4 + 4 + 4 + 2$

$= 16$

Number of arrangements that end with a multiple of 4 is $7! \times 16$ (1)

All other digits.

16 multiples of 4

$$P(\text{Divisible by } 4) = \frac{7! \times 16}{9!}$$

$$= \frac{16}{9 \times 8}$$

$$= \frac{2}{9} \quad (1)$$

(c) $W = 6000 - Ae^{-kt}$

When $t = 0$, $W = 91$

$t = 5 \quad W = 2300$

$\therefore \quad 91 = 6000 - A$

$A = 5909$ (1)

$$2300 = 6000 - 5909e^{-5k}$$

$$5909e^{-5k} = 3700$$

$$e^{-5k} = \frac{3700}{5909}$$

$$-5k = \ln\left(\frac{3700}{5909}\right)$$

$$k = \frac{\ln\left(\frac{3700}{5909}\right)}{-5}$$

$$\doteqdot 0.0936\ldots \quad 1$$

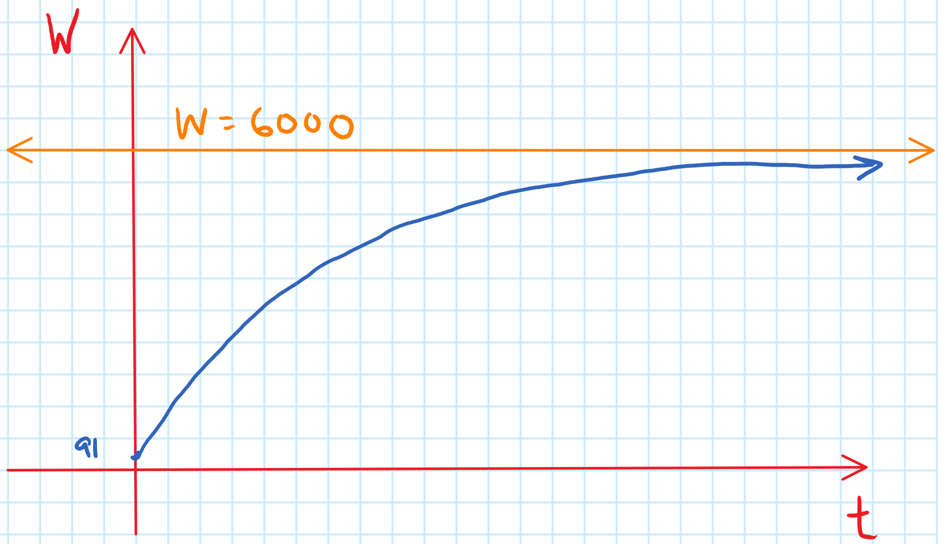
(ii) $$W = 6000 - Ae^{-kt}$$

$$\frac{dW}{dt} = kAe^{-kt}$$

When $t = 20$ $$\frac{dW}{dt} = kAe^{-k \times 20}$$

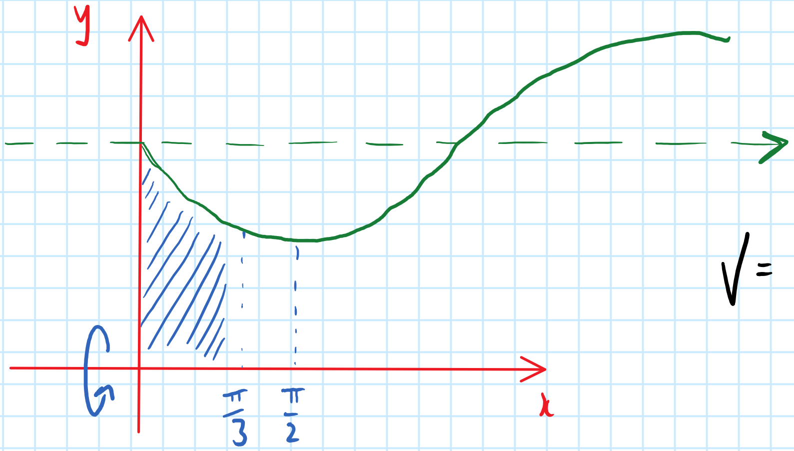
$$\doteqdot 85 \text{ kg/year} \quad 1$$

(iii)



1 Shape & Asymptote

(d)



$$V = \pi \int_a^b y^2 \, dx$$

$$V = \pi \int_0^{\frac{\pi}{3}} (3 - \sin x)^2 \, dx \quad 1$$

$$= \pi \int_0^{\frac{\pi}{3}} 9 - 6\sin x + \sin^2 x \, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\therefore V = \pi \int_0^{\frac{\pi}{3}} 9 - 6\sin x + \frac{1}{2} - \frac{\cos 2x}{2} \, dx \quad 1$$

$$\therefore V = \pi \int_0^{\frac{\pi}{3}} -6\sin x + \frac{19}{2} - \frac{\cos 2x}{2} \, dx$$

$$= \pi \left[\frac{19x}{2} + 6\cos x - \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{3}}$$

$$= \pi \left[\left(\frac{19\pi}{6} + 3 - \frac{\sqrt{3}}{8} \right) - (0 + 6 - 0) \right]$$

$$= \pi \left[\frac{19\pi}{6} - \frac{\sqrt{3}}{8} - 3 \right] \text{ units}^3$$

1

(a) $f(\theta) = \sin\theta - \cos\theta \qquad 0 \le \theta \le 2\pi$

(i) $t = \tan\frac{\theta}{2} \qquad \sin\theta = \frac{2t}{1+t^2} \qquad 0 \le \frac{\theta}{2} \le \pi$

$\cos\theta = \frac{1-t^2}{1+t^2}$

$\therefore \frac{2t - 1 + t^2}{1+t^2} = 1$

$2t - 1 + \cancel{t^2} = 1 + \cancel{t^2}$

$2t = 2$

$t = 1$ ①

$\therefore \tan\frac{\theta}{2} = 1$

$\frac{\theta}{2} = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$ ①

TEST $\theta = \pi$ As $\tan\frac{\pi}{2}$ is undefined

$\sin\pi - \cos\pi = 1$

$\therefore \theta = \pi$ is also a solution

$\therefore \theta = \frac{\pi}{2} \; \& \; \pi$ ①

(ii) There are two options

$\theta < \frac{\pi}{2}, \; \theta > \pi$ or $\frac{\pi}{2} < \theta < \pi$

TEST $\theta = 0$

$\sin 0 - \cos 0 = -1$

Which is < 1

$\therefore \theta < \frac{\pi}{2}, \; \theta > \pi$ ①

(b) Need the x coordinate of A

$\sin 2x = \cos x$

$\sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

$\therefore \quad x = \frac{\pi}{2} \quad \& \quad x = \frac{\pi}{6}$

$\therefore$ x coordinate of A is $\frac{\pi}{6}$ 1

$y = \sin 2x$

$\frac{dy}{dx} = 2\cos 2x$

$\therefore \quad m_1 = 2\cos\frac{\pi}{3}$

$= 1$ 1

$y = \cos x$

$\frac{dy}{dx} = -\sin x$

$m_2 = -\frac{1}{2}$ 1

let α be the angle between the tangents

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{3}{2}}{\frac{1}{2}}\right|$$

$\alpha = \tan^{-1} 3$ 1 $= 71°34'$ Nearest minute

Wednesday, 12 July 2017 9:18 AM

(c) (i) $x + py = 2ap + ap^3$

Meets the y axis when $x = 0$

$\therefore \quad y_Q = 2a + ap^2$

$= a(2 + p^2)$ 1

(ii) $P(2ap, ap^2) \qquad Q(0, a(2+p^2))$

$-2 : 1$

$R(x_R, y_R)$

$x_R = \dfrac{2ap}{-1}$

$= -2ap$

$y_R = \dfrac{ap^2 - 2a(2+p^2)}{-1}$

$= \dfrac{ap^2 - 4a - 2ap^2}{-1}$

$= 4a + ap^2$

$\therefore \quad R(-2ap, 4a + ap^2)$ As required

(iii) $x = -2ap \qquad y = 4a + ap^2$

$p = \dfrac{-x}{2a}$

$\therefore \quad y = 4a + a\left(\dfrac{x^2}{4a^2}\right)$

$= 4a + \dfrac{x^2}{4a}$ 1

Question 12, Page 4

Wednesday, 12 July 2017 9:23 AM

$$\frac{x^2}{4a} = y - 4a$$

$$x^2 = 4a(y - 4a)$$

1 This is a parabola with vertex $(0, 4a)$ & focal length a

(d) (i) When the first approximation is near a turning point.

(ii) $y = \sin^2 x + \ln(x-1)$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x) = \sin^2 x + \ln(x-1)$$

$$f'(x) = 2\cos x \sin x + \frac{1}{x-1}$$

$\therefore x_2 = -20.3027\ldots$ (A much worse approximation)

(a) (i) $^{12}C_1 \times {}^{11}C_1 \times {}^{10}C_4 \times {}^{6}C_3$ (1)

Pitcher Catcher Infield Outfield

$= 554400$ (1)

(ii) Total ways of NOT picking Raul & Petra in the outfield or infield

= Total teams (Part (i))

- Both in infield (B_I)

- Both in outfield (B_o) (1)

B_I Put both in the infield ∴ only 10 left.

$= {}^{10}C_1 \times {}^{9}C_1 \times {}^{8}C_2 \times {}^{6}C_3$

Pitcher Catcher 2 more infield Outfield

$= 50400$

B_o Put both in the outfield ∴ only 10 left.

$= {}^{10}C_1 \times {}^{9}C_1 \times {}^{8}C_4 \times {}^{4}C_1$

Pitcher Catcher Infield only one outfield

$= 25200$

∴ Total ways not in infield or outfield together $= 478800$ (1)

(b) $v = (16 - x)$ When $t = 0$ $x = 15$

$$\frac{dx}{dt} = 16 - x$$

$$\frac{dt}{dx} = \frac{1}{16-x}$$

$$\therefore t = -\ln(16-x) + c \quad 1$$

When $t = 0$, $x = 15$

$$0 = -\ln(1) + c$$

$$\therefore t = -\ln(16-x)$$

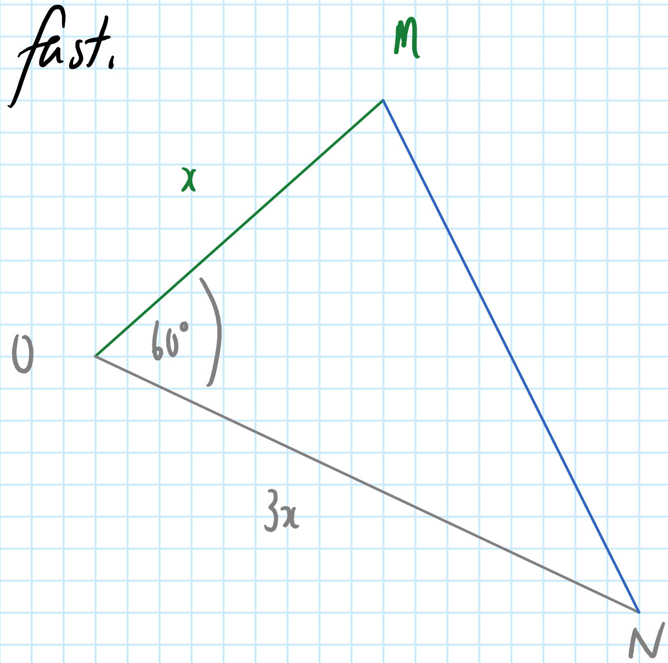
$$e^{-t} = 16 - x$$

$$x = 16 - e^{-t} \quad 1$$

(ii) As $t \to \infty$ $e^{-t} \to 0$

$\therefore x \to 16$

(c) If M travels x metres then N travels $3x$ metres as it is going 3 times as fast.



$$A = \frac{1}{2}ab \sin C$$
$$= \frac{1}{2} \times x \times 3x \times \sin 60^\circ$$
$$= \frac{3x^2}{2} \times \frac{\sqrt{3}}{2}$$
$$= \frac{3\sqrt{3}x^2}{4}$$ As required

(ii) $\frac{dx}{dt} = 15$ when $x = 100$

$$\frac{dA}{dt} = \frac{dx}{dt} \times \frac{dA}{dx}$$ 1

$$\frac{dA}{dx} = \frac{6\sqrt{3}\,x}{4}$$

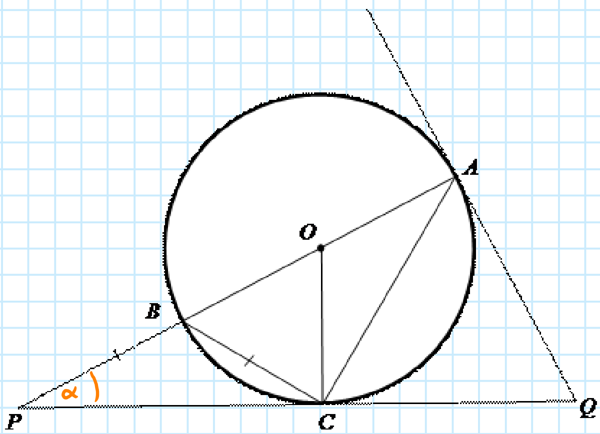
When $x = 100$

$$\frac{dA}{dx} = 150\sqrt{3}$$

$$\therefore \frac{dA}{dt} = 15 \times 150\sqrt{3}$$

$$= 2250\sqrt{3} \quad m^2/s$$ 1

(d)



(i) let $\angle BPC = \alpha$

$\angle BCP = \alpha$ (Base $\angle$'s in an isosceles Δ are equal)

$\angle BCA = \alpha$ ($\angle$ between a tangent & a chord is equal to the $\angle$ subtended in the opposite segment) 1

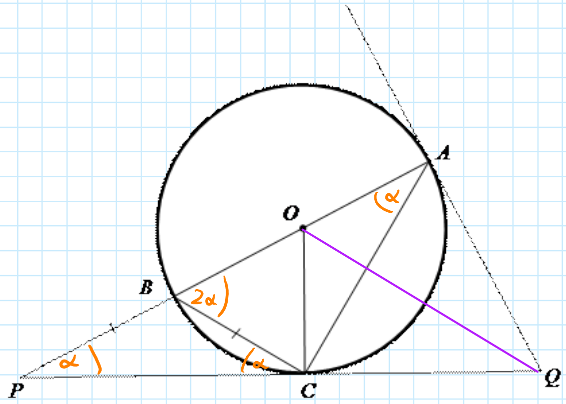
$\angle BCA = 90°$ ($\angle$ subtended by a diameter is $90°$)

$\angle ABC = 2\alpha$ (external $\angle$ in ΔBPC is equal to the sum of the interior opposites)

$\therefore \alpha + 90° + 2\alpha = 180°$ ($\angle$ sum of ΔABC)

$\therefore \alpha = 30°$ 1 correct show with reasons.

(ii)



$\angle OCA = \alpha$

(Base $\angle$'s of Isosceles ΔAOC are equal)

$\angle AOC = 180 - 2\alpha$

($\angle$ sum of ΔAOC is $180°$)

$OA = OC$ (radii of the same circle)

$QC = QA$ (tangents drawn from the same external point are equal)

$\therefore OCQA$ is a kite

$\angle OCQ = \angle OAQ = 90^\circ$ (radii meet tangents at 90° at the point of contact)

$\therefore \angle AQC = 2\alpha$ (Δ sum of quadrilateral $OAQC$)

$\angle OQC = \alpha$ (Diagonal of kite $OAQC$ bisects $\angle AQC$)

$\Delta POC \equiv \Delta QOC$

$\angle OPC = \angle OQC$ (see above)

$\angle OCP = \angle OCQ$ (radii meet tangents at 90° at the point of contact)

OC is common

$\therefore \Delta POC \equiv \Delta QOC$ (AAS)

① awarded for these statements.

① progress towards 3rd component of proof.

① complete cong. proof with reasons.

Wednesday, 12 July 2017 10:18 AM

(a) (i) $f(x) = \dfrac{9x^2 - 1}{x^4}$

$f(-x) = \dfrac{9(-x)^2 - 1}{(-x)^4}$

$= \dfrac{9x^2 - 1}{x^4}$

$= f(x)$ $\therefore$ even ①

(ii) $y = \dfrac{9}{x^2} - \dfrac{1}{x^4}$

$\dfrac{dy}{dx} = \dfrac{-18}{x^3} + \dfrac{4}{x^5}$

$= \dfrac{-18x^2 + 4}{x^5}$

$= \dfrac{-2(9x^2 - 2)}{x^5}$ ①

$\dfrac{d^2y}{dx^2} = \dfrac{54}{x^4} - \dfrac{20}{x^6}$

$= \dfrac{54x^2 - 20}{x^6}$

$= \dfrac{2(27x^2 - 10)}{x^6}$

Stationary points occur when $\dfrac{dy}{dx} = 0$

$9x^2 - 2 = 0$

$x = \pm \dfrac{\sqrt{2}}{3}$

When $x = \dfrac{\sqrt{2}}{3}$ $y = \dfrac{891}{4}$ $\dfrac{d^2y}{dx^2} = -729$ $\therefore$ Max ①

$x = -\dfrac{\sqrt{2}}{3}$ $y = \dfrac{891}{4}$ $\dfrac{d^2y}{dx^2} = -729$ $\therefore$ Max

Question 14, Page 2

Wednesday, 12 July 2017 10:18 AM

(iii) Points of inflexion occur when $\frac{d^2y}{dx^2} = 0$

$\therefore \quad 27x^2 - 10 = 0$

$x = \pm\sqrt{\frac{10}{27}}$ 1

x	-0.61	$-\sqrt{\frac{10}{27}}$	-0.6
$\frac{d^2y}{dx^2}$	$-1.81\ldots$	0	$12.00\ldots$

x	0.6	$\sqrt{\frac{10}{27}}$	0.61
$\frac{d^2y}{dx^2}$	$12.00\ldots$	0	$-1.81\ldots$

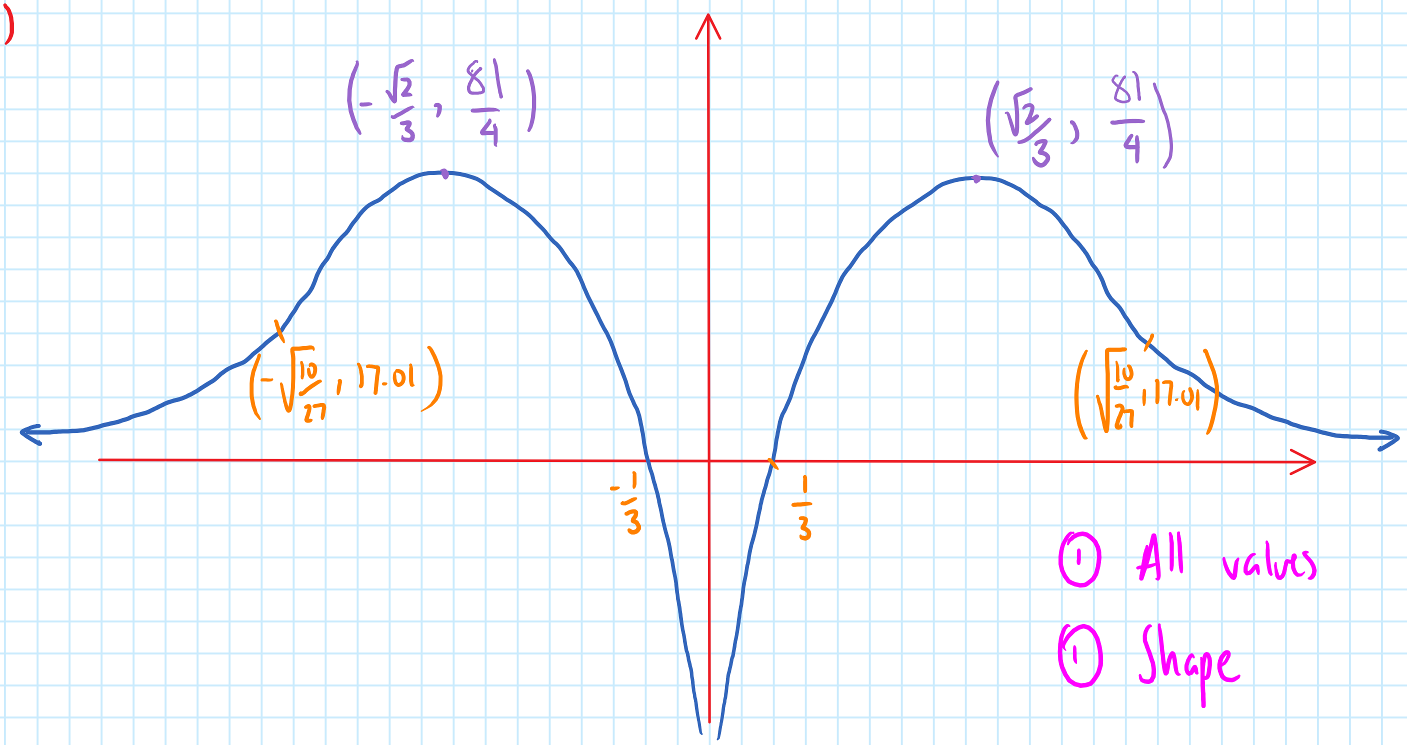
$\therefore$ Change in concavity 1 Test

$\therefore$ Points of inflection @ $\left(\sqrt{\frac{10}{27}}, 17.01\right)$ and $\left(-\sqrt{\frac{10}{27}}, 17.01\right)$

(iv) Vertical asymptote @ $x = 0$ 1

As $x \to \infty$ $y \to 0$ 1

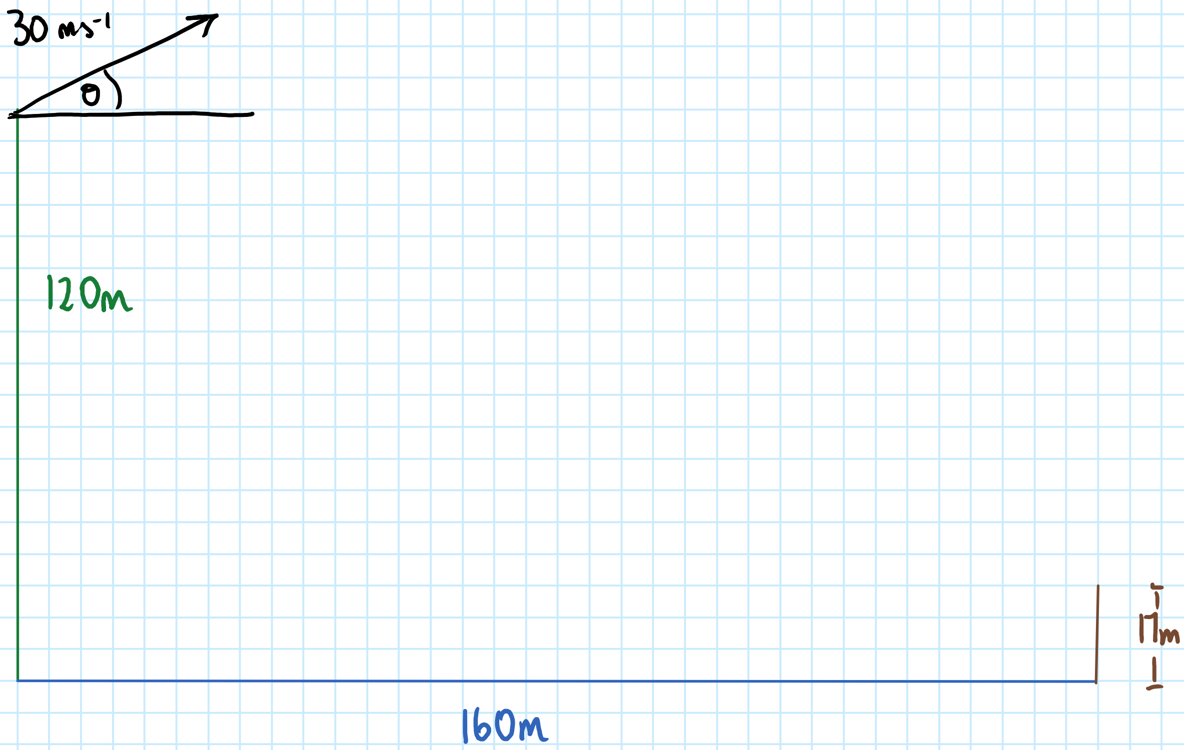
(v)



1 All values

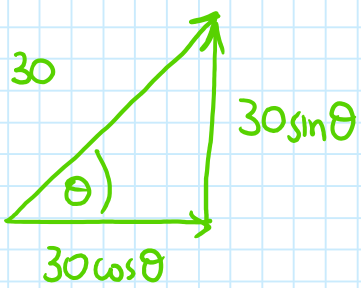
1 Shape

Wednesday, 12 July 2017 10:18 AM



(i) $\ddot{x} = 0$ $\qquad$ $\ddot{y} = -g$

$\dot{x} = +C_1$ $\qquad$ $\dot{y} = -gt + C_2$



1

$\therefore \dot{x} = 30\cos\theta$ $\qquad$ $\dot{y} = -gt + 30\sin\theta$

$x = 30t\cos\theta + C_3$ $\qquad$ $y = \frac{-gt^2}{2} + 30t\sin\theta + C_4$

When $t = 0$ $\quad x = 0$, $y = 120$ 1

$x = 30t\cos\theta$ $\qquad$ $y = -4.9t^2 + 30t\sin\theta + 120$

(ii) If $x = 30t\cos\theta$ $\qquad y = -4.9t^2 + 30t\sin\theta + 120$

$$t = \frac{x}{30\cos\theta}$$

$$y = -4.9\left(\frac{x^2}{30^2\cos^2\theta}\right) + 30\sin\theta \times \frac{x}{30\cos\theta} + 120 \quad (1)$$

$$y = \frac{-4.9}{30^2}x^2\left(\sec^2\theta\right) + \left(\tan\theta\right)x + 120$$

$$y = \frac{-4.9}{30^2}x^2\left(1 + \tan^2\theta\right) + 1\left(\tan\theta\right)x + 120$$

$$y = \frac{-4.9x^2}{30^2} - \frac{4.9x^2}{30^2}\tan^2\theta + x\tan\theta + 120$$

let $A = \tan\theta$

$$y = \frac{-4.9x^2}{30^2} - \frac{4.9x^2}{30^2}A^2 + xA + 120$$

$$\frac{4.9x^2}{30^2}A^2 - xA + y + \frac{4.9x^2}{30^2} - 120 = 0$$

$$\therefore A = \frac{x \pm \sqrt{x^2 - 4 \times \frac{4.9x^2}{30^2} \times \left(y + \frac{4.9x^2}{30^2} - 120\right)}}{2 \times \frac{4.9x^2}{30^2}} \quad (1)$$

Question 14, Page 5

Wednesday, 12 July 2017 10:18 AM

Need to solve for 2 positions

$M(160, 0)$ & $N(160, 17)$

@ M $A = 1.01\ldots$ & $0.137\ldots$

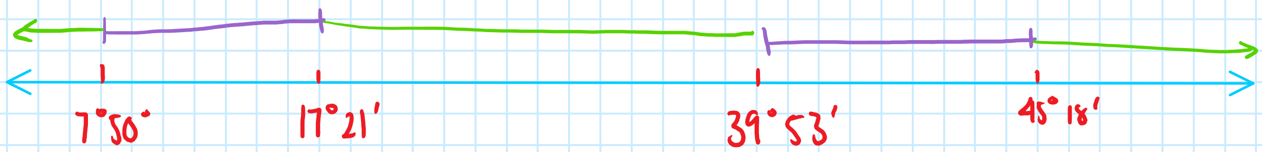
$45°18'$ & $7°50'$

@ N $A = 0.835\ldots$ & $0.312\ldots$

$39°53'$ & $17°21'$

1 All nearest minute

2 options



At a launch angle of 0° it does not reach the ship

$\therefore$ $7°50' \leq \theta \leq 17°21'$ & $39°53' \leq \theta \leq 45°18'$

1